Standing Waves
Waves in a string or a wire

- Support rod
- Wire or string
- Pulley
- Function generator
- Mechanical driver
- Clamp
- Mass
The reflected pulse is inverted and its amplitude is unchanged.
(a) Discontinuity where the wave speed increases

Before:

After:

String with smaller wave speed

String with larger wave speed

Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley
(b) Discontinuity where the wave speed decreases

Before:

After:
The reflected pulse is inverted.
Fundamental, first harmonic

Second harmonic

Third harmonic

Fourth harmonic

Fifth harmonic

\[
\begin{array}{cccc}
\text{n} & \lambda_n & f_n \\
1 & \frac{2L}{1} & 1 \frac{v}{2L} \\
2 & \frac{2L}{2} & 2 \frac{v}{2L} \\
3 & \frac{2L}{3} & 3 \frac{v}{2L} \\
4 & \frac{2L}{4} & 4 \frac{v}{2L} \\
5 & \frac{2L}{5} & 5 \frac{v}{2L} \\
\end{array}
\]
<table>
<thead>
<tr>
<th>$n$</th>
<th>$\lambda_n$</th>
<th>$f_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{4L}{1}$</td>
<td>$1 \frac{v}{4L}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{4L}{3}$</td>
<td>$3 \frac{v}{4L}$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{4L}{5}$</td>
<td>$5 \frac{v}{4L}$</td>
</tr>
<tr>
<td>7</td>
<td>$\frac{4L}{7}$</td>
<td>$7 \frac{v}{4L}$</td>
</tr>
<tr>
<td>9</td>
<td>$\frac{4L}{9}$</td>
<td>$9 \frac{v}{4L}$</td>
</tr>
</tbody>
</table>
Questions
Equation of Standing Wave

Consider two waves travelling in opposite directions

\[ D_1(x, t) = A \sin(kx - \omega t) \]
\[ D_2(x, t) = A \sin(kx + \omega t) \]

\[ D_{\text{Net}}(x, t) = ? \]

Recall trig identity

\[ \sin \theta_1 + \sin \theta_2 = 2 \cos \frac{\theta_1 + \theta_2}{2} \sin \frac{\theta_1 - \theta_2}{2} \]
\[ D_{net}(x,t) = 2A \cos(\omega t) \sin(kx) \]

For many reflections:

\[ D_{net}(x,t) = A_{net} \cos(\omega t) \sin(kx) \]

Amplitude oscillates

Stationary in space
String fixed at \(x = 0\) and \(x = L\)

At \(x = 0\), \(\sin(k \cdot 0) = \sin(0) = 0\)

At \(x = L\), want \(\sin(kL) = 0\) (Node)

Requires \(kL = n\pi\), \(n\) an integer

\[
(2\pi/\lambda)L = n\pi \quad \text{&} \quad v = \lambda f
\]

\[
f = nv/2L
\]
String fixed at $x = 0$ and open at $x = L$

At $x = 0$, $\sin(k*0) = \sin(0) = 0$ \checkmark

At $x = L$, want $\sin(kL) = 1$ (Antinode)

Requires $kL = m\pi/2$, $m$ an odd integer

$$(2\pi/\lambda)L = m\pi/2 \& \nu = \lambda f$$

$$f = m\nu/4L$$
Sound Level
Questions
Sound Intensity

Intensity $I = \text{Power/Area (W/m}^2\text{)}$

Spherical sound source:

$A = 4\pi R^2, \quad I = P / 4\pi R^2$

If $I = I_1$ at position 1,

$I_2 = I_1 / 2^2 = I_1 / 4$ since it is twice as far from source
Sound Level

Describes perception of intensity

$$\beta = 10 \log \frac{I}{I_0}, \quad I = I_0 \times 10^{\beta/10}$$

For multifrequency sound, intensity adds

$$I_{\text{net}} = I_1 + I_2 + ...$$

Note! $\beta_{\text{net}} \neq \beta_1 + \beta_2 + ...$
<table>
<thead>
<tr>
<th>Source</th>
<th>$I/I_0$</th>
<th>dB</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal breathing</td>
<td>$10^0$</td>
<td>0</td>
<td>Hearing threshold</td>
</tr>
<tr>
<td>Rustling leaves</td>
<td>$10^1$</td>
<td>10</td>
<td>Barely audible</td>
</tr>
<tr>
<td>Soft whisper (at 5 m)</td>
<td>$10^2$</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Library</td>
<td>$10^3$</td>
<td>30</td>
<td>Very quiet</td>
</tr>
<tr>
<td>Quiet office</td>
<td>$10^4$</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Normal conversation (at 1 m)</td>
<td>$10^5$</td>
<td>50</td>
<td>Quiet</td>
</tr>
<tr>
<td>Busy traffic</td>
<td>$10^6$</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Noisy office with machines; average factory</td>
<td>$10^7$</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Heavy truck (at 15 m); Niagara Falls</td>
<td>$10^8$</td>
<td>80</td>
<td>Constant exposure endangers hearing</td>
</tr>
<tr>
<td>Old subway train</td>
<td>$10^9$</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Construction noise (at 3 m)</td>
<td>$10^{10}$</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Rock concert with amplifiers (at 2 m); jet takeoff (at 60 m)</td>
<td>$10^{11}$</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>Pneumatic riveter; machine gun</td>
<td>$10^{12}$</td>
<td>120</td>
<td>Pain threshold</td>
</tr>
<tr>
<td>Jet takeoff (nearby)</td>
<td>$10^{13}$</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>Large rocket engine (nearby)</td>
<td>$10^{14}$</td>
<td>150</td>
<td></td>
</tr>
</tbody>
</table>