CHARACTERISTICS OF DRY FRICTION & PROBLEMS INVOLVING DRY FRICTION

Today’s Objective:

Students will be able to:

a) Understand the characteristics of dry friction.
b) Draw a FBD including friction.
c) Solve problems involving friction.
READING QUIZ

1. A friction force always acts _____ to the contact surface.
   A) normal  B) at $45^\circ$
   C) parallel  D) at the angle of static friction

2. If a block is stationary, then the friction force acting on it is ________.
   A) $\leq \mu_s N$  B) $= \mu_s N$
   C) $\geq \mu_s N$  D) $= \mu_k N$
APPLICATIONS

In designing a brake system for a bicycle, car, or any other vehicle, it is important to understand the frictional forces involved.

For an applied force on the brake pads, how can we determine the magnitude and direction of the resulting friction force?
APPLICATIONS
(continued)

Consider pushing a box as shown here.

How can you determine if it will slide, tilt, or stay in static equilibrium?

What physical factors affect the answer to this question?
Friction is defined as a force of resistance acting on a body which prevents or retards slipping of the body relative to a second body.

Experiments show that frictional forces act tangent (parallel) to the contacting surface in a direction opposing the relative motion or tendency for motion.

For the body shown in the figure to be in equilibrium, the following must be true: \( F = P, \quad N = W, \) and \( Wx = Ph. \)
CHARACTERISTICS OF DRY FRICTION

(continued)

To study the characteristics of the friction force $F$, let us assume that tipping does not occur (i.e., “$h$” is small or “$a$” is large). Then we gradually increase the magnitude of the force $P$. Typically, experiments show that the friction force $F$ varies with $P$, as shown in the right figure above.
The **maximum friction force** is attained just before the block begins to move (a situation that is called “impending motion”). The value of the force is found using \( F_s = \mu_s \, N \), where \( \mu_s \) is called the coefficient of static friction. The value of \( \mu_s \) depends on the materials in contact.

Once the block begins to move, the frictional force typically drops and is given by \( F_k = \mu_k \, N \). The value of \( \mu_k \) (coefficient of kinetic friction) is less than \( \mu_s \).
DETERMING $\mu_s$ EXPERIMENTALLY

A block with weight $w$ is placed on an inclined plane. The plane is slowly tilted until the block just begins to slip.

The inclination, $\theta_s$, is noted. Analysis of the block just before it begins to move gives (using $F_s = \mu_s N$):

\[
\begin{align*}
\sum F_y &= N - W \cos \theta_s = 0 \\
\sum F_x &= \mu_s N - W \sin \theta_s = 0
\end{align*}
\]

Using these two equations, we get $\mu_s = (W \sin \theta_s) / (W \cos \theta_s) = \tan \theta_s$

This simple experiment allows us to find the $\mu_s$ between two materials in contact.
Steps for solving equilibrium problems involving dry friction:

1. Draw the necessary free body diagrams. Make sure that you show the friction force in the correct direction (it always opposes the motion or impending motion).

2. Determine the number of unknowns. Do not assume \( F = \mu_s N \) unless the impending motion condition is given.

3. Apply the equations of equilibrium and appropriate frictional equations to solve for the unknowns.
IMPENDING TIPPING versus SLIPPING

For a given $W$ and $h$, how can we determine if the block will slide first or tip first? In this case, we have four unknowns ($F$, $N$, $x$, and $P$) and only three E-of-E.

Hence, we have to make an assumption to give us another equation. Then we can solve for the unknowns using the three E-of-E. Finally, we need to check if our assumption was correct.
IMPENDING TIPPING versus SLIPPING
(continued)

Assume: Slipping occurs

Known: \( F = \mu_s N \)

Solve: \( x, P, \) and \( N \)

Check: \( 0 \leq x \leq b/2 \)

Or

Assume: Tipping occurs

Known: \( x = b/2 \)

Solve: \( P, N, \) and \( F \)

Check: \( F \leq \mu_s N \)
EXAMPLE

**Given:** A uniform ladder weighs 20 lb. The vertical wall is smooth (no friction). The floor is rough and \( \mu_s = 0.8 \).

**Find:** The minimum force \( P \) needed to move (tip or slide) the ladder.

**Plan:**

a) Draw a FBD.

b) Determine the unknowns.

c) Make any necessary friction assumptions.

d) Apply E-of-E (and friction equations, if appropriate) to solve for the unknowns.

e) Check assumptions, if required.
There are four unknowns: $N_A$, $F_A$, $N_B$, and $P$. Let us assume that the ladder will tip first. Hence, $N_B = 0$

\[ + \uparrow \Sigma F_Y = N_A - 20 = 0 ; \quad \text{so } N_A = 20 \text{ lb} \]

\[ + \downarrow \Sigma M_A = 20 (3) - P(4) = 0 ; \quad \text{so } P = 15 \text{ lb} \]

\[ + \rightarrow \Sigma F_X = 15 - F_A = 0 ; \quad \text{so } F_A = 15 \text{ lb} \]
Now check the assumption.

\[ F_{\text{max}} = \mu_s N_A = 0.8 \times 20 \text{ lb} = 16 \text{ lb} \]

Is \( F_A = 15 \text{ lb} \leq F_{\text{max}} = 16 \text{ lb} \)? Yes, hence our assumption of tipping is correct.
EXAMPLE (slipping case)

There are four unknowns: $N_A$, $F_A = \mu_s N_A$, $N_B$, and $P$. Let us assume that the ladder will tip first. Hence, $N_B = 0$

\[ + \sum F_Y = N_A - 20 = 0 ; \quad \text{so} \quad N_A = 20 \text{ lb, } F_A = 16 \text{ lb} \]

\[ + \sum M_A = 20 \cdot 3 - P \cdot 4 - 8N_B = 0 ; \]

\[ + \sum F_X = N_B + P - 16 \text{lb} = 0 ; \quad \text{so } P = 17 \text{ lb } & N_B = -1\text{lb} \]
CONCEPT QUIZ

1. A 100 lb box with wide base is pulled by a force $P$ and $\mu_s = 0.4$. Which force orientation requires the least force to begin sliding?

A) $P(A)$  
B) $P(B)$  
C) $P(C)$  
D) Can not be determined

2. A ladder is positioned as shown. Please indicate the direction of the friction force on the ladder at B.

A) $\uparrow$  
B) $\downarrow$  
C) $\rightarrow$  
D) $\leftarrow$
GROUP PROBLEM SOLVING

Given: Drum weight = 100 lb, \( \mu_s = 0.5 \), a = 3 ft and b = 4 ft.

Find: The smallest magnitude of P that will cause impending motion (tipping or slipping) of the drum.

Plan:

a) Draw a FBD of the drum.

b) Determine the unknowns.

c) Make friction assumptions, as necessary.

d) Apply E-of-E (and friction equation as appropriate) to solve for the unknowns.

e) Check assumptions, as required.
There are four unknowns: P, N, F and x.

First, let’s assume the drum slips. Then the friction equation is $F = \mu_s \ N = 0.5 \ N$. 
GROUP PROBLEM SOLVING (continued)

A FBD of the drum:

\[ + \sum F_X = \left( \frac{4}{5} \right) P - 0.5 N = 0 \]
\[ + \sum F_Y = N - \left( \frac{3}{5} \right) P - 100 = 0 \]

These two equations give:

\[ P = 100 \text{ lb} \quad \text{and} \quad N = 160 \text{ lb} \]

\[ + \sum M_O = \left( \frac{3}{5} \right) 100 \,(1.5) - \left( \frac{4}{5} \right) 100 \,(4) + 160 \,(x) = 0 \]

Check: \( x = 1.44 \leq 1.5 \) so OK!

Drum slips as assumed at \( P = 100 \text{ lb} \)
GROUP PROBLEM SOLVING (tipping case)

There are four unknowns: P, N, F and x.

First, let’s assume the drum tips. Then the x = 1.5 ft.
GROUP PROBLEM SOLVING (continued)

A FBD of the drum:

\[ + \rightarrow \sum F_X = \frac{4}{5} P - F = 0 \]

\[ + \uparrow \sum F_Y = N - \frac{3}{5} P - 100 = 0 \]

\[ \sum M_{\text{Edge}} = \frac{3}{5} P \cdot 3 - \frac{4}{5} P \cdot 4 + 100 \cdot 1.5 = 0 \]

So \( P = 107 \text{ lb} \). Check: \( F = 86 \text{ lb} \) and \( N = 164 \text{ lb} \), so \( F_{\text{max}} = 0.5(164 \text{ lb}) = 82 \text{ lb} < F \) found therefore not possible it has slipped.
ATTENTION QUIZ

1. A 10 lb block is in equilibrium. What is the magnitude of the friction force between this block and the surface?
   A) 0 lb  B) 1 lb  
   C) 2 lb   D) 3 lb

2. The ladder AB is positioned as shown. What is the direction of the friction force on the ladder at B.
   A)  B)  
   C) ←   D) ↑